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ABSTRACT

The nature and use of the Cornell Mathematics Support Center (MSC) is discussed. It is viewed as significantly different from other existing support programs in that it is part of the Department of Mathematics and offers encouragement and support to any interested student, no matter what that pupil's mathematics level is. The MSC is based on the belief that mathematics can become a part of every person's understanding and on the following principles: (1) A person deserves to have confidence in his/her ability to understand and think about mathematics; (2) It is not a person's fault when that confidence is lost and it is always possible to regain it; and (3) Learning and thinking about mathematics does not have to be done in isolation. A 12-cell matrix is presented that explicates student concerns in the three categories: (1) concerns of closeness and isolation; (2) context and meaning; and (3) power and powerlessness in the context of mathematics learning and teaching. The bulk of the document focuses on three cells, using students as examples, to outline some primary observations. The report concludes that mathematics is a way of seeing, and that students come to the MSC for help in expressing what they see. (MP)

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FINDINGS AND IMPLICATIONS FROM THE MATHEMATICS SUPPORT CENTER AT CORNELL UNIVERSITY

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FINDINGS AND IMPLICATIONS FROM THE MATHEMATICS SUPPORT CENTER

AT CORNELL UNIVERSITY

> "I know that I am smart, but when my professor said he didn't have time to help me catch up then I panicked! I did not think I could learn it without any help."
>
> "I don't want to memorize. I can follow what my professor did but I want a way to visualize, I want to see it for myself."
>
> "I am a little puzzeled. Lately my advisor has been handing back my writing and there are comments on it like "Very nice here." or "Good idea." He has never done anything like this before and it seems strange but I like it. He is doing it to encourage me and I am grateful."

These are comments from three different visitors to the Cornell Mathematics Support Center, the setting for this study. Each comment represents a category of concerns students have with respect to their mathematical learning experience. The purpose of this paper is to point out these categories and to indicate their powerful and subtle nature.

The Cornell Mathematics Support Center (MSC) differs significantly from other existing mathematics support programs in that it is part of the Department of Mathematics and that it offers encouragement and support to any student interested in mathematics, no matter at what level of mathematics the student is. The intention is that the MSC be intimately part of the Department of Mathematics. The idea is that anyone involved in mathematics runs into difficulty at some time or another and needs to talk to someone. Other support centers often focus on students in need of remedial work or 'developmental courses' (Thum). The MSC is open to these students and offers some course-tutoring for any student. But the large majority of the students who visit the MSC are regular students in regular courses.

Students come for a variety of reasons. Some are A students who seek help because they feel they are not really understanding some mathematics. Often they are seeing something in the mathematics that is different from what is discussed in their class. As a result fears develop. The students have doubts about the value of their own perceptions and of the validity of their own mathematical knowledge as a background against which to make mathematical speculations and claims.

A surprise is that the graduate students in the Mathematics Department are frequent visitors to the MSC. They come to talk through a mathematics problem or practice a seminar talk or seek career information. They come to pour out feelings of frustration, fears of not being creative enough to be a research mathematician and hesitancies about their commitment to mathematics. Also, teaching assistants, those graduate students responsible for teaching recitation sections, find the MSC a place to discuss their ambivalent feelings about teachers, from their position as students who also teach.

In addition to working directly with the students who come to the MSC, the MSC acts as a mathematics promotion center and offers seminars, art and mathematics symposiums, workshops and other events desitned to stimulate interest in mathematics and teaching and interaction between faculty, undergraduate students, graduate students, staff and general community.

Many people (including students who are taking mathematics courses) learn and view mathematics in rigid, rote ways that lock them into conditioned responses that limit their creativity (Collins, 1979). Many students are afraid that they cannot understand mathematics. The MSC is based on the belief that mathematics can become a part of every person's understanding and on the principles:

1) A person deserves to have confidence in his or her ability to understand and think about mathematics.
2) It is not a person's fault when that confidence is lost and it is always possible to regain it.
3) Learning and thinking about mathematics does not have to be done in isolation.

The students were the focus of attention in the MSC and in this study more than the mathematics. The atmosphere of the MSC was made safe for the students and was egalitarian, non-hurried and private. Students simply were asked, "How may I be of help to you?" or "What would you like to have happen right now?" or "What is going on for you?" The responses to this open-ended invitation revealed commonalities in the concerns students have; commonalities that hinge on the desire to make and share mathematical meaning and on the significance of feelings. The explicit presentation of these commenalities was the focus of the study.

The theory of educating of philosopher and educator D. Bob Gowin, the theory of intellectual and ethical development described by Willian Perry, Jr. and his associates at Harvard University, and the possibility of a "women's view of mathematics" form the background for the interpretation of the data. Gowin's model was chosen for this study because it gives attention to the importance of integrating thinking, feeling and acting. In addition, its acknowledgement of students' claims to what they can bring to the educating episode as well as the 'give and take' quality in Gowin's definition of teaching as shared meaning gives sanction to the student's efforts to make personal meaning in mathematics.

An especial strength of Gowin's theory for this study lies in its articulation of educational events into four commonplaces of teacher, student, curriculum, and social governance. These commonplaces were used

as a framework by which to organize the data. The concerns of students were grouped into three broad and overlapping categories: concerns of closeness and isolation, context and meaning, and power and powerlessness in the context of mathematics learning and teaching. Thus, this study explicates student concerns in terms of the elements of the following matrix:

	Feelings of closeness and isolation	Feelings of personal meaning and context	Feelings of power and powerlessness
Teacher			
Student			
Curriculum			
Governance			

A complete description of this matrix or its twelve cells is too lengthy for this paper. Instead, part of the contents of three of the cells will be used to outline some primary observations.

1. Gifted Students Need Support (From Closeness and Isolation/ Teacher.)

Students feel a need to see the faculty; that is, to know how faculty makes decisions, chooses what mathematics to teach, what examples to present, how handle uncertainties about mathematics. And there is a reciprocal need to be seen; to be recognized as bringing something to the teaching episode. A surprise was that some gifted students felt isolated from the faculty. Example: Lori was a high school student who became behind in her mathematics as a result of work with her school play and a later trip with parents. She came to the MSC with a sheaf of mimeographed notes about common logarithms with the aid of which she said she was expected to work out homework on linear interpolation. Whether or not the mimeographed pages were the total extent of the teacher's assistance, the student believed that that was all that was forthcoming and took on the responsibility of asking about the MSC and coming up the hill to the campus. Nowhere on the sheets was there a

graph of a log curve and nowhere was any connection overtly made between lines and linear interpolation. Without interpretation from the teacher, the student could make little sense of the ratio technique for interpolating and didn't understand what was expected or accomplished. Yet, when she was given the coordinates of two points, told that they lie on the same straight line and then was given the abscissa of an intermediate point and asked to find the ordinate, she did not need to be told how to proceed. She quickly developed an accurate method for determining the coordinate. Then,when I helped her make a graph of $y = \log x$ and note its linear qualities, she readily understood the assumptions that form the basis of linear interpolation.

This example shows the necessity for context in learning and for ascertaining what a student already knows, reminding the student of it, and teaching accordingly (Ausubel, Novak) but also it shows that gifted students need support. It has long been thought that a teacher who conveys the expectation that students are able to do their work will have that expectation fulfilled. The effect on some students, however, is less one of encouragement due to a conveyance of positive expectation of student achievement (although that is what the teacher may intend) than it is feelings of abandonment, of being pushed away into the category of gifted students with the assumption that gifted students don't need help.

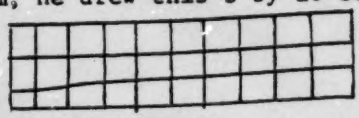
Isolation from faculty may be more pronounced for gifted female students than for male students. Becker did an observation of 100 geometry classes and found that teachers seemed unaware of a bias to involve male students in more interactions of all kinds. The teachers seemed to expect females to be more conscientious in their work and not need extra attention. A study by the Fennema Group at Wisconsin (Reyes) showed that high confidence females had the least amount of interaction with their high school teachers.

It is possible that some gifted students hold back from giving feedback to their teachers because they don't want to interrupt what is going on in

their classrooms. (One interpretation of Matina Horner's research on women's "fear of success " is that women do not fear success as much as they responsibly avoid competition. (Sassen.)) Patricia Lund Casserley interviewed high school girls who qualified for advanced placement programs in mathematics, chemistry or physics. Her study suggests that perhaps gifted students feel guilty for being born with so much; or perhaps they feel over-anxious about their responsibility to use their God given talents. They are made to feel they shouldn't take up teacher time and that they should be able to manage on their own.

2. Students Want to Make Personal Meaning in Mathematics (From Personal Meaning and Context/ Curriculum)

The quality of questions brought to the MSC by the students showed that students want to trust and use their past experience when doing present mathematics. Often they came with their homework done correctly but said, "I did all these problems but it was mechanical. I felt like a robot." They came with ideas but were hesitant about developing them or sharing them. Many students have lost confidence that knowledge they learn on their own is as worthwhile as that which they can find in a book or hear in a lecture. Example: An ex-marine, used to using a hand-held calculator, asked about a problem discussed in his pre-calculus class. The problem read, "A real-estate investor had a 30 acre piece of land and planned to subdivide it into building sites of 2/3 acre each. How many building sites will that be?" The student seemed hesitant as to what to do but when encouraged to look at the problem any way that made sense to him, he drew this 3 by 10 subdivision:



With a determined air the student then divided some of the 'acres' into thirds, began to count, and arrived at the number of 45 sites. He said, "Well, that is what the answer sheet has," but he still seemed puzzled. Then I noticed

a calculation he had made in his notes. It seems his original strategy was to take 1.33 x 30 = 39.9 (thinking of 1-1/3 sites per acre.) Once he realized that there were 1-1/2 sites/acre his strategy worked fine. This student had looked at this problem in two quite different but valid ways. His first thought was that there would be more than 30 sites. Then he used the idea of 1-1/3 sites/acre but the answer sheet said he was five short. He re-analyzed the problem to solve it by actual counting. This person is using good problem solving techniques: estimate, work the problem, check, reformulate and work again. He was doubting his intelligence however, as he pointed to what was written on the mimeographed answer sheet: $\frac{30}{2/3} = 30 \times 3/2 = \frac{90}{2} = 45$, and said, "I would never have thought of that." The student said he was feeling like a total failure until he received feedback on his approaches from the MSC.

Example: Student "B" is a calculus student who has tried calculus before but either failed or dropped out. He is determined to be a physics major and so is taking the course again. He is a regular visitor to the MSC but rarely talks about his course. Mostly he would talk about mathematical notions inspired by other seminars or books. Being able to talk about these tangential mathematical ideas seemed to be as or more helpful as talking about the specific content of his course. Here I pressed him to look at his calculus exercises.

B: I really would not know how to do one of these proofs. I would need a great deal of prompting each time. Here is one problem. Prove that,... um,...o.k....looks something like that. So it would appear that they are asking you to prove nonsense.

MSC: What are they asking you to prove?

B: They are asking you to prove that zero is the only critical point and that f(0) is the extreme. (Some discussion) So if it's zero (first derivative) it's telling you that at that point the slope is zero. But if it is zero, it is telling you that at that point it is an extremum or a point of inflection unless it was a point of discontinuity.

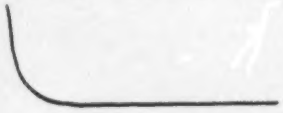
M: Is it possible for it to be zero and not be a maximum, minimum or inflection point?

B: Well, unless it were an asymptote but...

M: If it is going to go up, it is going to have to change concavity. There is no other way. That would have to be a cusp there...

B: (Excited) A cusp? A sharp point? If you just didn't change that what would you have if you just came down from zero. That would be the only exception then, because that wouldn't really be quite a local maximum or minimum but the derivative certainly would be defined. He didn't mention this in lecture or I maybe missed it.

This is what the student drew on the board:



What the student was saying was that here is an ambiguous or borderline case for the first derivative test. Listening to students revealed that when they did or said something that seemed strange or wrong it was not necessarily wrong. Sometimes students asked questions or made remarks that appeared irrelevant but which established connections for them. To help students value their ideas, it was important to listen to them. If they are not listened to, then how are they to know that they are being taken seriously as creators of mathematical knowledge?

Students and teachers may have very different notions about the meaning of some mathematical concept (Davidson). It may be difficult or scary for a teacher in the classroom to recognize that a student is seeing something

different in the mathematics (Henderson) expecially since commonly it is assumed that the teacher should be 'on top of things'. Also, teachers may not hear the questions students ask because they are concentrating on the mathematics and not on the students understanding of the mathematics. The students who came to the MSC did not come to be told how to do homework, they came for support for their own efforts to make meaning in the mathematics.

3. **How One Feels Plays a Significant Part in Thinking About Mathematics (From Feelings of Power and Powerlessness/ Student.)**

The feeling that something is wrong with one can reduce one's confidence in ability and inhibit clear thinking. Sometimes mathematics students felt they were being looked upon as different or weird. Some students panicked when they felt they had been left alone to figure everything out without help. Negative feelings can overwhelm students so that they cannot think about mathematics. But it was not having the feelings that caused this difficulty. It was not being aware of them.

Students who were in touch with their feelings found ways to work towards success. Students who allowed themselves to feel their intuitions then could follow their intuitions, the resultant exploration often leading to new insights into the mathematics (See also Buerk.) Students who acknowledged feelings of elation when something 'clicked' used that high as inspiration to keep on with their work. Often students reported being discouraged or intimidated by the course material, the tests, the professor or the university. If they ignored or did not pay attention to what they were feeling, then it was possible for negative feelings to overwhelm them and leave them mentally paralyzed.

Example: A first year calculus student talked about her homework and tests. She was upset but thought about her feelings and spoke about them articulately.

> I always feel like I'm wrong. I panic. I remember something like this one or I remember what the answer looks like. If I can kind of change it a little bit then I'd get a similar answer. All this is not really conscious. But then I find I had that answer, the correct one. I did. But I changed it.

The student describes how, when she panics as on a test, she tries to rely on memory of similar problems and answers rather than on her own thinking. She realizes that she must make sense of the problem for herself, that her feelings influence her thinking and that if she were aware of the negative feelings just a bit sooner, she would not be overwhelmed by them.

> I feel it is wrong so it is wrong. Panic makes me rely on what I feel and on memory. When I start to panic I think I allow my mind to go blank before it quite does. Then when it's blank, it is like a snowy TV screen and I can't think of anything.

This student didn't always feel so powerless.

> Before I came here, in high school, I thought I was smart. Then when I came, I began to realize that I might not be the smartest...then, maybe not even in the upper average. Then I think even that I'm really just dumb!

As this student talked, she began to realize that her feelings were not necessarily an accurate reflection of her abilities and that she did not have to act or rely on them. She saw that she did not have to succumb to her panic and that she could take a position of powerfulness over her feelings. I suggested that she say to herself, "I have a good mind." whenever the panic feelings threatened and she found that these reminders helped her keep her panic feelings at bay.

Example: Barbara was taking entrance tests to determine whether or not she would be accepted into the adult program in a college. She read the followirg quotation from Margaret Mead and reacted in terms of her feelings about mathematics. First is the quotation of Margaret Mead.

> Resistence in the human sciences to treating the human being as a whole person is very great. Not because the human sciences are soft or easy--they are much harder--but because they call for a kind of empathy. It's not detachment; it's knowing who you are and who the other person is, watching both, and using the interaction between the two as one of your tools...much more involvement in introspection and of empathy is needed if we are going to understand what's happening.

Barbara circled the words 'watching both' and described her fears and how she is able to 'rise above the emotion' and modify them.

> My mind (ego) tends to 'protect itself' when danger or threats come along. "I can't do this math!" My mind finds math a threat and adrenaline shoots out; my palms sweat. It is only by watching the interaction will I be able to rise above the emotion and modify my fears.
>
> > "Yes, my palms are sweaty. Now I'll do the math with sweaty palms. It is o.k. to do math with sweaty palms. I'm an o.k. person even if I have problems with math."
>
> What helps me is to give myself permission to have fear. (Obviously it is watching the interaction.) "It's o.k., Barbara, to have sweaty palms. You are still a loveable person." I don't know whether it is the permission or the watching that allows me to 'get a handle' on the situation.

Barbara is very clear that it is after she is aware of her fears that she is able to find ways to take power over them. Ignoring feelings of confusion can result in very strange looking actions on the part of students; as when a calculus student concluded that both 1 and -1 were the answer to $f'(1/x)$ at $x = 1$ (also see Confrey). Ignoring feelings can result in near paralysis of thinking. But students who acknowledged their feelings were able to find ways to take power over them.

Our answer to "What is mathematics?" may be the crucial component determining how accessible mathematics appears to us. "Do you think that if I eat, sleep and breathe mathematics that I will become a mathematician?" This is a serious question posed by a senior mathematics major. Behind the question is an

assumption. The assumption is that mathematics is an entity onto itself, an austere and beautiful monolith which one can attempt to scale or one can stand in awe, at its feet. (See Shelly for this analogy.) The myth that mathematics is remote, a closed system cut in stone, inhibits people from believing that they can think about mathematics and see mathematics in the world. This myth is perpetuated when mathematics is presented as a gleaming, polished product devoid of human experience.

Example: One of the most frequent visitors to the MSC was a student who often talked in a philosophical vein. Once, while he was doing an integration problem I heard him mutter, "If I have an equation and take the derivative of both sides of the equation then I still will have an equation; but that has to be taken on faith."

I was quite surprised to hear him say this and I concurred saying, "Yes, it is a leap of faith." My saying this shocked him and he demanded to know all about me, how much mathematics I had had and if I really was a math person. He said, "No mathematician would ever see that!"

Those teachers who let the students see that the teacher has had to grapple with the ideas will reassure them that mathematics is confusing and that it is alright to feel confused.

Example: One of the very best graduate students in the mathematics department spent almost a year being very depressed about his abilities. He spoke often about his doubts that he could ever be creative enough to be a good research mathematician:

> "It takes years to get to the forefront of an area in mathematics. Only a few are creative enough to do good research in mathematics."

But the crux of his anguish went deeper:

> "The faculty has some way of dealing with their confusion. They have some way where they don't let it bother them when they don't understand something. I don't know how they are able to shrug it off and keep going."

This student is pointing painfully to the need for the faculty to share their uncertainties and how they resolve their uncertainties for themselves with the students.

For those students who had made a commitment to mathematics, often the feeling of identity with mathematics brought a sense of isolation from others or from one's own total personality.

Example: An advanced mathematics graduate student stood in front of me with tears in his eyes and said, "What do you do if you are 80 - 90% mathematics? If you've let yourself become consumed by mathematics so that that is what you are. And then you want to let someone get to know you. What do you do when you can't explain that much of yourself to them? How can they get to know you when they can't understand that much of you?"

For many graduate students working intimately with mathematics there is a tension between the pull of the mathematics and a desire to preserve one's "real self" or one's "own personality" as being more than the mathematics. There are teachers who invite the students to use the relevance of their life experiences to pose and explore situations in the mathematics and to reflect on the feelings experienced in the process. (Brown, Copes). For these teachers, mathematics is a way of understanding our universe, and their entire course may be generated by the exploration of a concept that we use to describe our universe. Mathematics is not a problem to be defined and solved. It is not a book or a set of techniques. Students are looking for ways to broaden and deepen their experience of life. Mathematics is a way of seeing. The students come to the MSC for help in expressing what they see.

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